Set theory symbols are used to express relationships and operations between sets.

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| --- | --- | --- |
| symbol |  | explanation |
|  | Subset | This symbol means “is a subset of”.  If , it means every element of set is also an element of set . It includes the possibility that could be equal to .  if and only if |
|  | Proper Subset | Similar to , but with the difference that  if , it means every element of set  is also an element of set ,  but is not equal to |
|  | Superset | This symbol means "is a superset of.  If , it means every element of set is also an element of set . It includes the possibility that could be equal to . |
|  | Proper Superset | Similar to , but with the difference that  if , it means every element of set  is also an element of set ,  but is not equal to |
|  | Not a Subset | This symbol indicates that a set is not a subset of another set.  For example, if , it means there exists at least one element in set that is not in set |
|  | Not a Superset | This symbol indicates that a set is not a superset of another set.  For example, if , it means there exists at least one element in set that is not in set . |
|  | Empty Set | This symbol represents the set with no elements, also known as the null set. |
|  | Union | The union of two sets and , denoted by , is the set containing all elements that are in or in , or in both. |
|  | Intersection | The union of two sets and , denoted by , is the set containing all elements that are in and |
|  | Set Difference | The set difference of two sets and , denoted by or , is the set containing all elements of that are not in . |

represents set membership

If is an element of , it doesn't necessarily mean that is a subset or superset of . It simply means that is one of the elements contained within the set .

Powersets

A power set is a mathematical concept that relates to sets and their subsets.

Given a set , the power set of , denoted by , is the set containing all possible subsets of , including the empty set and itself.

For example, let's say you have a set . The power set would contain all possible combinations of elements from , including the empty set and itself:

Where:

is the empty set

and are singleton sets containing only one element from .

is the set containing all elements of .

Collectively, these subsets make up the power set .

The cardinality (number of elements) of the power set of a set with elements is , including the empty set and the set itself.

Vector Spaces

**Vector Language: The axioms for a vector space**

A vector space is a set of vectors with addition and scalar multiplication that satisfy the axioms of addition and scalar multiplication.

In order for a set V to be a vector space the following 10 axioms must be true:

|  |  |  |
| --- | --- | --- |
| **A1** | **Closure under addition**  *For any vectors and in the set, is also in the set* |  |
| **A2** | **Existence of an additive identity**  *There exists a vector in the set such that for any vector in the set, .*  *Related:*  *- A3:Existence of additive inverses* |  |
| **A3** | **Existence of additive inverses**  *For every vector in the set, there exists a vector in the set such that* |  |
| **A4** | **Associativity of addition**  *For any vectors , , and in the set,*  *fundamental property of addition* |  |
| **A5** | **Commutativity of addition**  *For any vectors and* *in the set,*  *fundamental property of addition* |  |
| **M1** | **Closure under scalar multiplication**  *For any scalar and any vector in the set, is also in the set.*  *Implied by:*  *- A1:**Closure under addition* |  |
| **M2** | **Distributive property - vector addition**  *For any scalar and any vectors and in the set, +cv.*  *Implied by:*  *- A1:**Closure under addition*  *- A4:* *Associativity of addition*  *- A5:**Commutativity of addition* |  |
| **M3** | **Distributive Property - scalar addition**  *For any scalars and and any vector in the set, +cv.*  *Implied by:*  *- A1:**Closure under addition*  *- A4:* *Associativity of addition*  *- A5:**Commutativity of addition* |  |
| **M4** | **Associative Property**  (Compatibility of scalar multiplication with field multiplication)  *For any scalars and and any vector in the set, +cv.*  *Implied by:*  *- A1:**Closure under addition*  *- A4:* *Associativity of addition*  *- A5:**Commutativity of addition* |  |
| **M5** | **Multiplicative identity**  *For any vector in the set, , where 1 is the multiplicative identity of the underlying field*.  *Implied by:*  *- A1:**Closure under addition*  *- A4:* *Associativity of addition*  *- A5:**Commutativity of addition* |  |

**ASS2: [**Problem 5] (1)

*Determine whether each set equipped with the given operation is a vector space.*

*For those that are not vector space identify the vector space axioms that fail.*

*(1) The set with the standard operations on*

**Ref for proofs:** [**https://home.cc.umanitoba.ca/~borgerse/14R-T1/MATH2300/Worksheets/VectorSpaceExamples.pdf**](https://home.cc.umanitoba.ca/~borgerse/14R-T1/MATH2300/Worksheets/VectorSpaceExamples.pdf)

*(no Axioms* *failed for this question, this is just an example to show how to do this)*

|  |  |
| --- | --- |
| A1 | Let .  Then      Therefore A1 holds. |
| A2 | Let  the zero vector in be  Then      Therefore A2 holds. |
| A3 | Let  Then      Therefore A3 holds. |
| A4 | Addition in follows the same rules as addition in , so associativity holds.  OR  Let  Then |
| A5 | Addition in follows the same rules as addition in , so commutativity holds. |

Since all 5 fundamental axioms for vector spaces are satisfied,

equipped with the standard operations on  is indeed a vector space.

|  |  |
| --- | --- |
| **M1** | Let  Then      Therefore M1holds. |
| **M2** | Let  Then      Therefore M2holds. |
| **M3** | Let  Then            Therefore M3holds. |
| **M4** | Let  Then            Therefore M4holds. |
| **M5** | Let  Then        Therefore M5holds. |